Unit 8: Uncertainty in AILH 3



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• Outline:

- 8.1 Fuzzy sets
- 8.2 Fuzzy logic
- 8.3 Fuzzy inferences
- 8.4 Probability theory and uncertainty

Introduction to Fuzzy Logic

- The word **fuzzy** refers to things which are not clear or are vague.
 - E.g., tall, small, good,.... etc.
- Any event, process, or function that is changing continuously cannot always be defined as either true or false
- such activities can be defined in a Fuzzy manner by using fuzzy logic.

• What is Fuzzy Logic?

- fuzzy logic (by Lofti Zadeh in 1965) is not logic that is fuzzy, but logic

that is used to describe fuzziness.

- i.e., It deals with vague and imprecise information.
- based on degrees of truth rather than usual true/false or 1/0 like Boolean

logic.

• Example:



- Why Fuzzy Logic?
 - Fuzzy logic is useful for commercial and practical purposes.
 - It can control machines and consumer products.
 - It may not give accurate reasoning, but acceptable reasoning.
 - Fuzzy logic helps to deal with the uncertainty in engineering.

Crisp set

- A crisp set is an unordered collection of different elements.
- Represented by listing all the elements comprising it.
- E.g.,
 - Set of vowels in English alphabet, $A = \{a, e, i, o, u\}$
 - Set of odd numbers less than $10, B = \{1,3,5,7,9\}.$
- In crisp set element is either the member of a set or not. i.e., membership of elements is either 100%(1) or 0% (0).
- Membership of element x in set A is:

$$f_A(x) = \begin{cases} 1, \text{ if } x \in A \\ 0, \text{ if } x \notin A \end{cases}$$

- Example of Crisp sets (set of tall men): if Height >180 then men is tall
- membership on crisp set based on above rule is as shown in table below:

Name	Height, cm		Degree of	Crisp Sets	,	•
		Crisp	Membership			
Chris	208	1	1.0			· · · · · · ·
Mark	205	1	0.8 -			
John	198	1			,	
Tom	181	···· 1 : · ·	. U.O -			
David	179	0	0.4 - • • • • • • • •			
Mike	172	<mark>0</mark> :	0.2	· · ·		
Bob	167	0 :	0.2		,	
Steven	158	0	0.0	·····	<u> </u>	<u> </u>
Bill	155	· · · <mark>0</mark> · · · ·	150 160		190	210
Peter	152	0			,	Height, cm

• So we can say that crisp set has exact boundary.

Fuzzy set

- Elements are allowed to be partially included in the set.
- i.e., A Fuzzy Set has Fuzzy Boundaries
- i.e., membership of elements can be 100%(1) to 0%(0).
- Membership of element x in set A is:

 $\mu_A(x) = \begin{cases} 1, \text{ if } x \text{ is totally in A} \\ 0, \text{ if } x \notin A \\ 0 < \mu A(x) < 1 \text{ if } x \text{ is partly in A.} \end{cases}$

• The classical example in fuzzy sets is tall men. The elements of the fuzzy set

"tall men" are all men, but their degrees of membership depend on their

height.

		Degree of Membership			
Name	Height, cm	Crisp	Fuzzy		
Chris	208	1	1.00		
Mark	205	1	1.00		
John	198	1	0.98		
Tom	181	1	0.82		
David	179	0	0.78		
Mike	172	0	0.24		
Bob	167	0	0.15		
Steven	158	0	0.06		
Bill	155	0	0.01		
Peter	152	0	0.00		



• In our "tall men" example, we can obtain fuzzy sets of tall, short and average men.



• a man who is 184 cm tall is a member of the average men set with a degree of membership of 0.1, and at the same time, he is also a member of the tall men set with a degree of 0.4. 12

Fuzzy Set Operations

- Fuzzy union (∪): the union of two fuzzy sets is the maximum (MAX) of each element from two sets.
- E.g.
 - $A = \{1.0, 0.20, 0.75\}$
 - $B = \{0.2, 0.45, 0.50\}$
 - $A \cup B = \{MAX(1.0, 0.2), MAX(0.20, 0.45), MAX(0.75, 0.50)\}$

 $= \{1.0, 0.45, 0.75\}$

- Fuzzy intersection (∩): the intersection of two fuzzy sets is just the MIN of each element from the two sets.
- E.g.
 - $A = \{1.0, 0.20, 0.75\}$
 - $B = \{0.2, 0.45, 0.50\}$
 - $A \cap B = \{MIN(1.0, 0.2), MIN(0.20, 0.45), MIN(0.75, 0.50)\} = \{0.2, 0.20, 0.50\}$

- Complement (_^c):
 - The complement of a fuzzy variable x is (1-x).
 - The complement of a fuzzy set is composed of all elements' complement.
 - Example.
 - $A = \{1.0, 0.20, 0.75\}$
 - $A^{c} = \{1 1.0, 1 0.2, 1 0.75\} = \{0.0, 0.8, 0.25\}$

Fuzzy rules

• A fuzzy rule can be defined as a conditional statement in the form:

IF x is A THEN y is B

- where x and y are linguistic variables(Height, Weight, etc); and
- A and B are linguistic values (tall, short, medium, etc).

- A classical IF-THEN rule uses binary logic, for example,
 - Rule1:IF speed is > 100 THEN stopping_distance is >10
 - Rule 2: IF speed is < 40 THEN stopping_distance is 2
 - The variable speed can have any numerical value between 0 and 220 km/h, and the variable stopping_distance can take value >=0.

- If than rules in a fuzzy form:
 - Rule: 1 IF speed is fast THEN stopping_distance is long
 - Rule: 2 IF speed is slow THEN stopping_distance is short
 - In fuzzy rules, the linguistic variable speed has values slow, medium and fast. The linguistic variable stopping_distance has values short, medium and long.

- In a fuzzy system, all rules fire to some extent, or in other words they fire partially.
- If the antecedent is true to some degree of membership, then the consequent is also true to that same degree.

• IF height is *tall* THEN weight is *heavy*



- A fuzzy rule can have multiple antecedents, for example:
 - IF project_duration is long AND project_staffing is large AND project_funding is inadequate
 - THEN risk is high

IF service is excellent OR food is delicious
THEN tip is generous

• The consequent of a fuzzy rule can also include multiple parts,

for instance:

– IF temperature is hot

THEN hot_water is reduced AND cold_water is increased

Fuzzy Inference Systems

• It has four main parts as shown in figure below:



- Fuzzification Module:
 - It transforms the system inputs, which are crisp numbers, into fuzzy input.
- Knowledge Base:
 - It stores IF-THEN rules provided by human experts.
- Inference Engine:
 - It simulates the human reasoning process by making fuzzy inference on the inputs and IF-THEN rules.
- Defuzzification Module:
 - It transforms the fuzzy set obtained by the inference engine into a crisp value.

- Fuzzy inference:
- The inference operations upon fuzzy IF–THEN rules performed by FISs are:
 - Compare the input with the membership functions on the antecedent part to obtain the membership values of each linguistic label. (this step is often called fuzzification.)
 - 2. Combine the membership values on the antecedent part to get firing strength (degree of fullfillment) of each rule.
 - 3. Generate the qualified consequents for each rule depending on the firing strength.
 - 4. Aggregate the qualified consequents to produce a crisp output. (This step is called defuzzification.)

Application Areas of Fuzzy Logic

• The key application areas of fuzzy logic are as given :

Automotive Systems

- Automatic Gearboxes
- Four-Wheel Steering
- Vehicle environment control

Consumer Electronic Goods

- Photocopiers
- Video Cameras
- Television

Domestic Goods

- Microwave Ovens
- Refrigerators
- Toasters
- Vacuum Cleaners
- Washing Machines

Environment Control

- Air Conditioners/Dryers/Heaters
- Humidifiers

Example of a Fuzzy Logic System

• Let us consider an air conditioning system as fuzzy logic system. This system adjusts the temperature of air conditioner by comparing the room temperature and the target temperature value.



- Advantages of FLSs:
 - Mathematical concepts within fuzzy reasoning are very simple.
 - You can modify a FLS by just adding or deleting rules due to flexibility of fuzzy logic.
 - Fuzzy logic Systems can take imprecise, distorted, noisy input information.
 - Fuzzy logic is a solution to complex problems in all fields of life, including medicine, as it resembles human reasoning and decision making.

- Disadvantages of FLSs
 - There is no systematic approach to fuzzy system designing.
 - They are understandable only when simple.
 - They are suitable for the problems which do not need high accuracy.

Probability versus Fuzzy

- Probability:
 - Probability is a formal examination of likelihood that an event will occur
 - Measured in terms of ratio of number of expected occurrence of the total number of possible occurrence.
 - Event class is crisply defined only to determine its randomness.
- Fuzzy:
 - Is a type of deterministic uncertainty
 - Measures membership which event occurs.
 - Interprets partial degree of truth
 - Support many valued logic having truthness or falsity from[0, 1]
 - Uses fuzzy set membership function.

Homework

- What is fuzzy logic? How it differ from traditional logic?
- What is fuzzy set how it differ from tradition set? Explain the basic set operations.
- What is fuzzy rules. Explain the difference between fuzzy rules and crisp rule with suitable example.
- What is fuzzy inferencing system? How inferencing is done in fuzzy systems.
- What is fuzzy logic? Explain the different application area of fuzzy logic and fuzzy systems.
- What is probability? How it differ from fuzzy.

Thank You !

